Fractions and Recurring Decimals

You might think that a decimal is just a decimal. But oh no — things get a lot more juicy than that...

Recurring or Terminating...

1) Recurring decimals have a pattern of numbers which repeats forever, e.g. $\frac{1}{3}$ is the decimal 0.333333... Note, it doesn't have to be a single digit that repeats. You could have, for instance: 0.143143143....
2) The repeating part is usually marked with dots or a bar on top of the number. If there's one dot, then only one digit is repeated. If there are two dots, then everything from the first dot to the second dot is the repeating bit. E.g. $0.2\dot{5} = 0.2555555...$, $0.\dot{2}\dot{5} = 0.25252525...$, $0.\dot{2}5\dot{5} = 0.255255255...$
3) Terminating decimals are finite (they come to an end), e.g $\frac{1}{20}$ is the decimal 0.05.

The denominator (bottom number) of a fraction in its simplest form tells you if it converts to a recurring or terminating decimal. Fractions where the denominator has prime factors of only 2 or 5 will give terminating decimals. All other fractions will give recurring decimals.

	Only prime factors: 2 and 5				Also other prime factors			
FRACTION	$\frac{1}{5}$	$\frac{1}{125}$	$\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{7}$	$\frac{1}{35}$	$\frac{1}{3}$	$\frac{1}{6}$
EQUIVALENT DECIMAL	0.2	0.008	0.5	0.05	$0.\dot{1}4285\dot{7}$	$0.0\dot{2}8571\dot{4}$	$0.\dot{3}$	$0.1\dot{6}$
	Terminating decimals				Recurring decimals			

For prime factors, see p5.

Converting terminating decimals into fractions was covered on the previous page.
Converting recurring decimals is quite a bit harder — but you'll be OK once you've learnt the method...

Recurring Decimals into Fractions

1) Basic Ones

Turning a recurring decimal into a fraction uses a really clever trick. Just watch this...

EXAMPLE: Write $0.\dot{2}3\dot{4}$ as a fraction.

1) Name your decimal — I've called it r. — Let $r = 0.\dot{2}3\dot{4}$
2) Multiply r by a power of ten to move it past the decimal point by one full repeated lump — here that's 1000: — $1000r = 234.\dot{2}3\dot{4}$
3) Now you can subtract to get rid of the decimal part:

$$\begin{array}{rl} 1000r &= 234.\dot{2}3\dot{4} \\ -\quad r &= 0.\dot{2}3\dot{4} \\ \hline 999r &= 234 \end{array}$$

4) Then just divide to leave r, and cancel if possible: — $r = \frac{234}{999} = \frac{26}{111}$

The 'Just Learning the Result' Method:

1) For converting recurring decimals to fractions, you could just learn the result that the fraction always has the repeating unit on the top and the same number of nines on the bottom...
2) BUT this only works if the repeating bit starts straight after the decimal point (see the next page for an example where it doesn't).
3) AND some exam questions will ask you to 'show that' or 'prove' that a fraction and a recurring decimal are equivalent — and that means you have to use the proper method.