Bounds

Rounding and bounds go hand in hand, and not just because they sort-of rhyme...

Upper and Lower Bounds

Whenever a measurement is rounded to a given UNIT, the actual measurement can be anything up to HALF A UNIT bigger or smaller.

EXAMPLE: A room is 9 m long to the nearest metre. Find upper and lower bounds for its length.

The actual length could be half a metre either side of 9 m.

lower bound = 8.5 m
upper bound = 9.5 m

Note that the actual value is greater than or equal to the lower bound but less than the upper bound. In the example above, the actual length could be exactly 8.5 m, but if it was exactly 9.5 m it would round up to 10 m instead. Or, written as an inequality (see p39), 8.5 m $\leq$ actual length $<$ 9.5 m.

EXAMPLE: The mass of a cake is given as 2.4 kg to the nearest 0.1 kg. What are the upper and lower bounds for the actual mass of the cake?

The rounding unit here is 0.1 kg, so the actual value could be anything between 2.4 kg ± 0.05 kg.

lower bound = 2.4 – 0.05 = 2.35 kg
upper bound = 2.4 + 0.05 = 2.45 kg

Maximum and Minimum Values for Calculations

When a calculation is done using rounded values there will be a DISCREPANCY between the CALCULATED VALUE and the ACTUAL VALUE:

EXAMPLES:

1. A floor is measured as being 5.3 m by 4.2 m, to the nearest 10 cm. Calculate minimum and maximum possible values for the area of the floor.

The actual dimensions of the floor could be anything from 5.25 m to 5.35 m and 4.15 m to 4.25 m.

Find the minimum area by multiplying the lower bounds, and the maximum by multiplying the upper bounds.

minimum possible floor area = 5.25 × 4.15
= 21.7875 m^2

maximum possible floor area = 5.35 × 4.25
= 22.7375 m^2

2. a = 5.3 and b = 4.2, both given to 1 d.p. What are the maximum and minimum values of a ÷ b?

First find the bounds for a and b. → $5.25 \leq a < 5.35$, $4.15 \leq b < 4.25$

Now the tricky bit... The bigger the number you divide by, the smaller the answer, so:

max(a ÷ b) = max(a) ÷ min(b)

and min(a ÷ b) = min(a) ÷ max(b)

max. value of a ÷ b = 5.35 ÷ 4.15
= 1.289 (to 3 d.p.)

min. value of a ÷ b = 5.25 ÷ 4.25
= 1.235 (to 3 d.p.)

Bound, bound, get a bound, I get a bound...

This is bound to come up in the exam — or at least, it's not beyond the bounds of possibility that it could. When you think you know this page, try an Exam Practice Question:

Q1 x and y are measured as 2.32 m and 0.45 m, both to the nearest 0.01 m.
a) Find the upper and lower bounds of x and y. [2 marks]
b) If $z = x + 1/y$, find the maximum and minimum possible values of z. [2 marks]